# Fair Termination for Parameterized Probabilistic Concurrent Systems

(TACAS'17)

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9 May 2019 (MOSCA'19)

# Outline

- Parameterized probabilistic concurrent systems

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- Regular model checking

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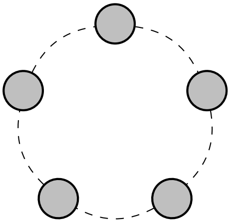

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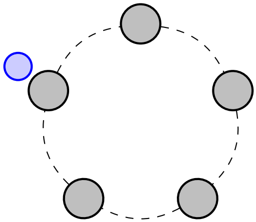

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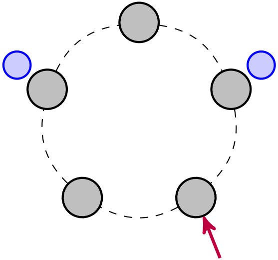

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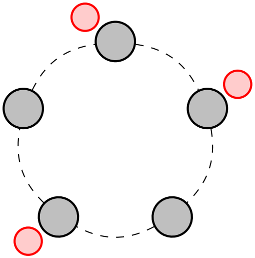

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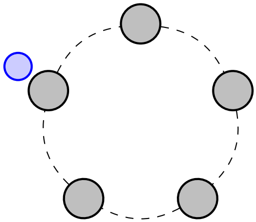

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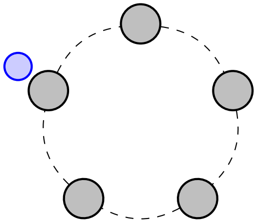

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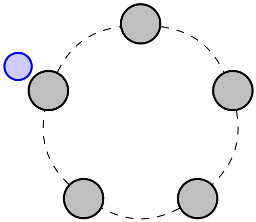

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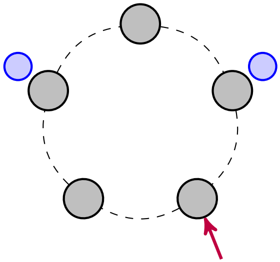

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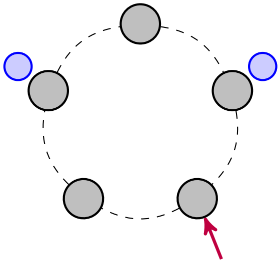

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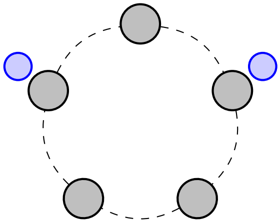

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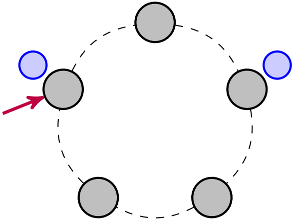

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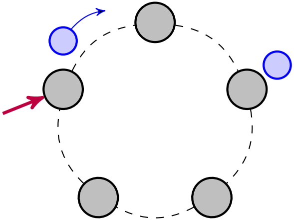

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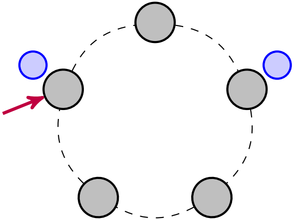

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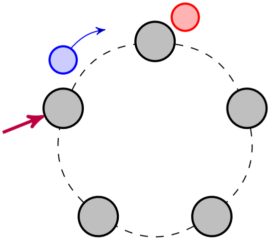

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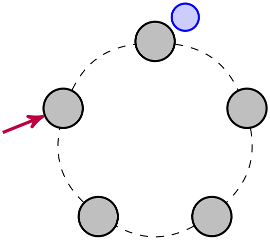

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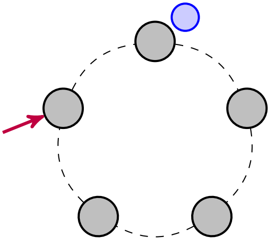

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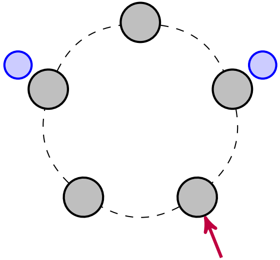

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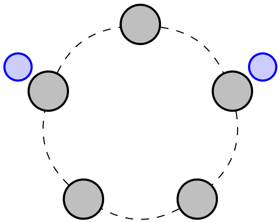

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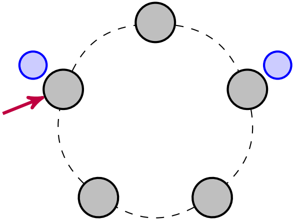

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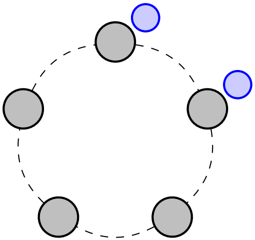

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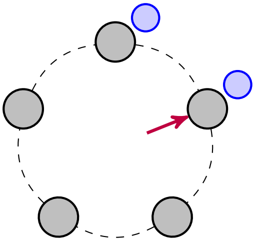

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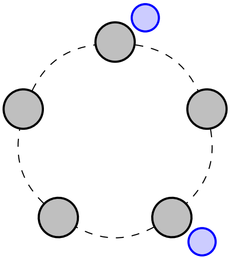

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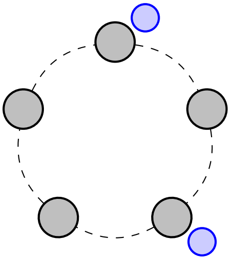

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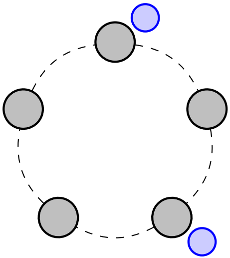

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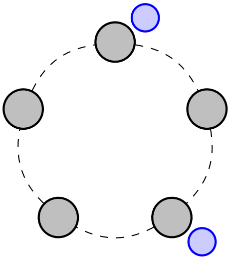

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- We use **finitary** fairness

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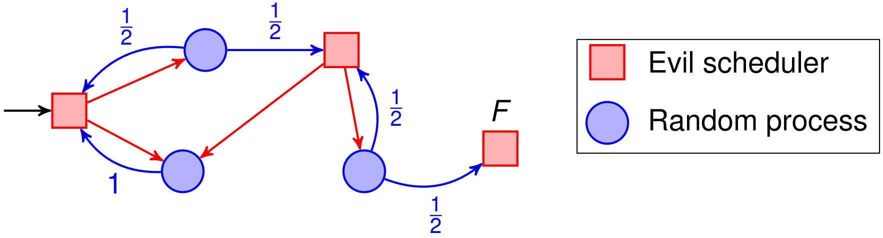

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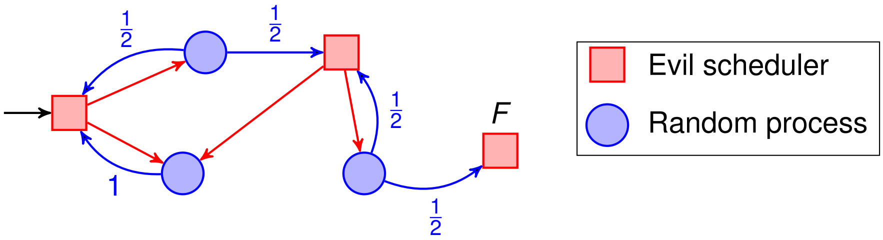


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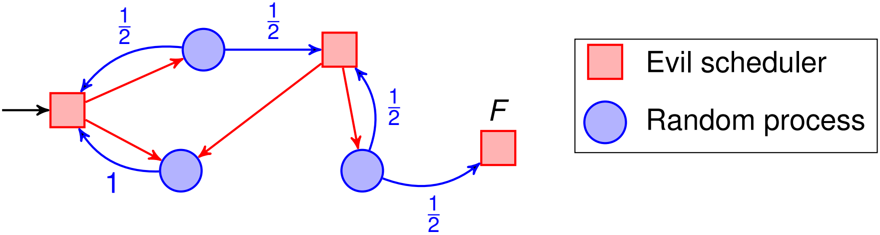

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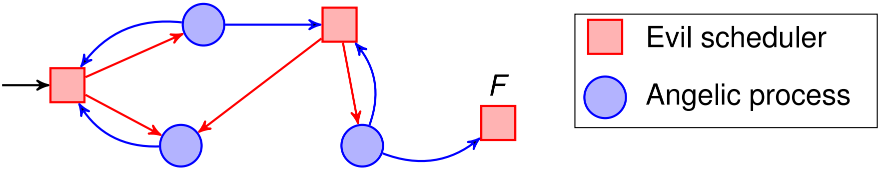

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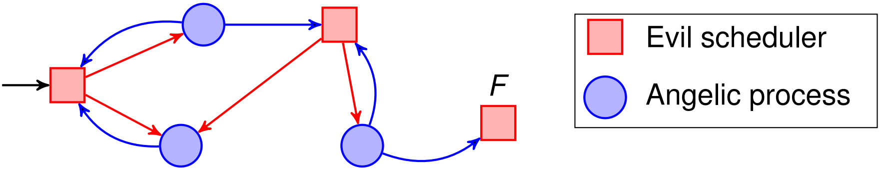


**Lemma**

$\Pr(s_0 \models \Diamond F) = 1$ *iff Proc. has winning strategy from all* $s \in Reach(s_0)$.

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- **this talk**: embedding of **fairness** into the system

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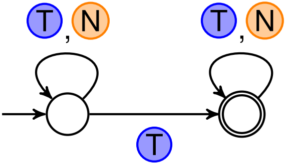

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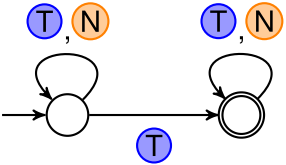
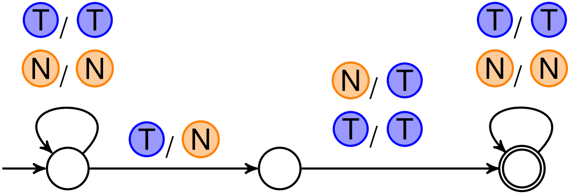



- Transition relation: a (length-preserving) **transducer** $\tau$



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  3. $P_<$ is a strict preorder (i.e., irreflexive, transitive)

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**Regular Model Checking** for 2-player reachability games:

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- *Start*, *Good*, $\tau_1$, and $\tau_2$ given
- **Advice bits**: local conditions on FA *Inv* and transducer $P_<$ over $\Sigma$
  1. $Start \subseteq Inv$
  2. $\tau_\cup(Inv) \subseteq Inv$
  3. $P_<$ is a strict preorder (i.e., irreflexive, transitive)
  4. For any evil transition from $Inv \setminus Good$ to $s_e$, there is an angelic transition from $s_e$ that
     - goes to *Inv* and
     - progresses w.r.t. $P_<$

$$\forall x \in Inv \setminus Good, \quad \forall y \in \Sigma^* \setminus Good :$$
$$(x \rightarrow_{\tau_1} y) \Rightarrow (\exists z \in Inv : (y \rightarrow_{\tau_2} z \wedge z <_P x))$$

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- Generalized to arbitrary weak and strong fairness

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- process player decrements/resets counters

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# Encoding Finitary Fairness into RMC

**Theorem**

*Let $S$ be a regular representation of an MDP with finitary fairness constraints $C$. The presented transformation yields a regular representation of an MDP $S_F$ (without fairness constraints) such that (if $C$ are realizable)*

$$\Pr(\mathit{Start} \models \Diamond \mathit{Good}) = 1 \qquad \mathit{iff} \qquad \Pr(\mathit{Start}_F \models \Diamond \mathit{Good}_F) = 1$$

# Case Studies: Population Protocols

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- **Cell cycle switch** — similar, but has an intermediate state

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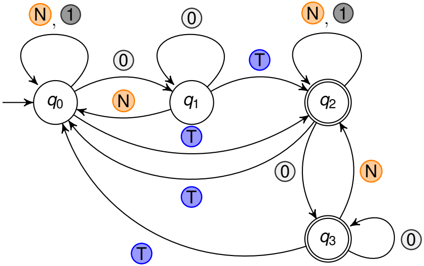
  - VERIFY: check the candidate is OK/refine SAT formula

# Evaluation

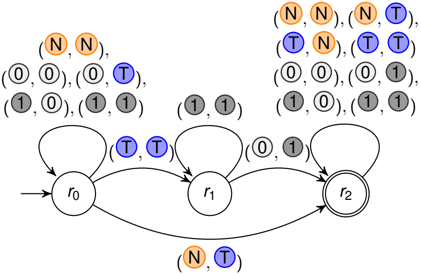
Table: Results of experiments (timeout = 10 hours).

| Case study | Time |
|---|---|
| Herman's protocol (merge, line) | 3.64 s |
| Herman's protocol (annih., line) | 4.33 s |
| Herman's protocol (merge, ring) | 4.31 s |
| Herman's protocol (annih., ring) | 4.61 s |
| Moran process (2 types, line) | 2 m 48 s |
| Moran process (3 types, line) | 56 m 14 s |
| Cell cycle switch (1 types, line) | 43.94 s |
| Cell cycle switch (2 types, line) | 9 h 46 m |
| Clustering (2 types, line) | 10 m 30 s |
| Clustering (3 types, line) | T/O |
| Coin game ($k = 3$, clique) | 1 m 0 s |

# Solution to Herman's protocol (merge, ring)




*Inv*




$P_<$

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